

Problem 1 (answer on page 1 of the booklet)

Find the domain of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 1} + \ln(4 - x^2 - y^2 - z^2)$. Determine if the domain of f is an open region, a closed region or neither? Also, determine if the domain is bounded or unbounded. Also find the equation of the level curve through the point $(1,1,0)$. (8 pts)

Problem 2 (answer on page 2 of the booklet)

Find the equations of the tangent plane and normal line to the surface $e^{xy} + \cos(xz) - \arctan(yz) + \frac{\pi}{4} = 2$ at the point $(0,1,1)$. (20 pts)

Problem 3 (answer on page 3 of the booklet)

Use the method of Lagrange multipliers to find the absolute maximum and minimum values of $f(x, y, z) = z - x^2 - y^2$ subject to the constraints $x + y + z = 1$ and $x^2 + y^2 = 4$. (20 pts)

Problem 4 (answer on page 4 of the booklet)

For each of the following limits, say if it exists or no, justifying your answer. (7+8+8 pts)

a) $\lim_{(x,y) \rightarrow (1,0)} \frac{(x-1)^2 \ln x}{(x-1)^2 + y^2}$ b) $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin x \sin y}{\sin^2 x + \sin^2 y}$ c) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^4 + 3y^4}$

Problem 5 (answer on pages 5 and 6 of the booklet)

Suppose that the directions of zero change of a function $f(x, y, z)$ at the point $(1,1,0)$ are $\vec{i} - \vec{j}$ and $-\vec{i} + \vec{j}$. Suppose also that the derivative of the function $f(x, y, z)$ increases most rapidly at the point $(2,0,1)$ in the direction of $A = 2i + j - k$ and the value of derivative at this point is $2\sqrt{6}$. Also suppose that

$f(1,1,0) = 3$, $f(3,1,4) = 2$, $f(2,0,1) = 6$ and $\nabla f(3,1,4) = 3i - 2j + k$.

Let $x = r + s$, $y = r - s$, $z = r^2 s$ and $w = f(x, y, z)$

- (i) Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ at $(r, s) = (1,0)$ then estimate $w(1.1, -0.05)$. (8 pts)
- (ii) Find the derivative of f at $(3,1,4)$ in the direction of $i + j - 4k$. (4 pts)
- (iii) Find a line normal to the surface $w(r, s) = 3e^{rs} + \ln r$ in the rs - plane. (8 pts)
- (iv) Find a plane tangent to the surface $w(r, s) = 5 + e^t$ in the rst - plane. (9 pts)